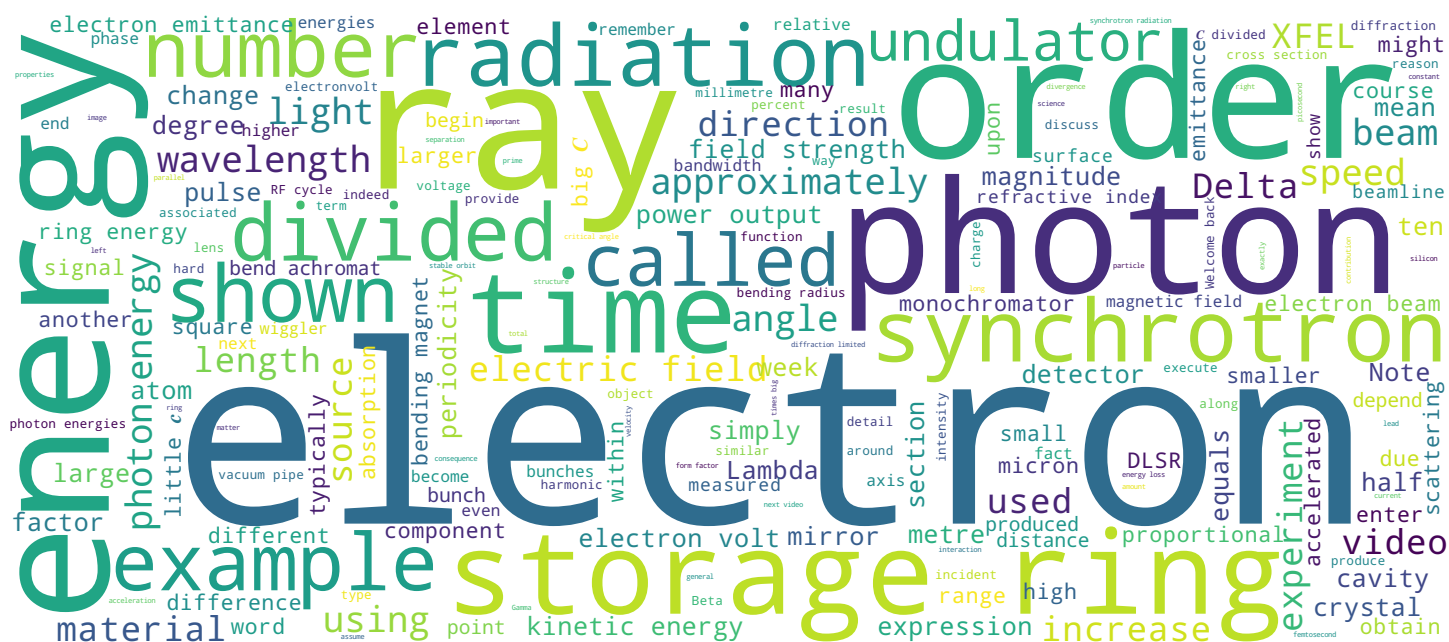


Synchrotrons and x-ray free-electron lasers

Techniques and applications

Prof. Philip Willmott



Search MOOC



Video



Contents and objectives of this video



- Radiated power from bending magnets
- Energy stored in storage rings
- The RF power supply
- RF bunching

Welcome back. In this video, we will consider the power radiated by any given synchrotron facility, considering only the radiation emitted by the bending magnets. In addition, the stored energy within the ring itself is very substantial because emission of photons by the accelerated electrons obviously implies that these electrons must lose energy. This loss of energy must be regularly replenished for the electrons to remain in a stable orbit. This will lead us to the beating heart of all storage rings, namely, the RF power supply. I use the term "beating" judiciously as a consequence of replenishing the electrons kinetic energy is to separate them into individual packets or bunches, with separation's equal to the periodicity of the RF supply measured in nanoseconds. And they have durations approximately 100 times shorter than this, measured in tens of picoseconds.

Notes

Summary



0m 05s

Radiated power from bending magnets

- Example: MAX IV, Lund

$$P[\text{kW}] = 1.266 \mathcal{E}^2[\text{GeV}] B^2[\text{T}] L[\text{m}] I[\text{A}]$$

$$\begin{array}{ccccccc} 1.266 & 3^2 = 9 & 0.5^2 = 0.25 & 2\pi\rho & 0.5 \text{ A} & & \\ & \underbrace{\hspace{1.5cm}} & & = 250 \text{ m} & & & \\ & \rho = 40 \text{ m} & & & & & \end{array}$$

$$= 356 \text{ kW}$$



2021 Tesla S Plaid, 1150 bhp

- Total radiated power larger
 - Insertion devices
 - Other magnet lattice components (small)
 - ~ + 500 kW



The equation for the radiated power of an arc sector of length " \mathcal{L} " is shown here without derivation. It depends on the square of both the storage ring energy and the dipole field strength, and linearly, with the average electron beam current. If we take as an example, the first DLSR to come online, Max 4, its storage ring energy of 3 gigaelectron volts, and dipole field strength of half a tesla lead to a bending radius of 40 metres, which in turn leads to a total length of the curved path of 250 metres, resulting in a power output from the dipole magnets alone around the entire ring of nearly 360 kilowatts, or the brake horsepower of a [inaudible 00:02:12] supercar. This number does not account for the power output from undulators and wigglers, which, depending on the facility, can be of a similar order of magnitude again as that produced by them bending magnets alone. Corrector magnets such as quadrupoles and sextupoles, generate only very weak power outputs as the field strength in their geometric centres is zero. And this is where ideally we would like the electron beam to pass through. The total power output is, therefore, closer to that of the upcoming Tesla S Plaid.

Notes

Summary



1m 15s

Stored energy in a storage ring

- Current = $I \sim 0.5 \text{ A}$
- Circumference = $C \sim 500 \text{ m}$
- Time to execute one orbit = $C/c \sim 2 \mu\text{s}$
- Number of stored electrons = $IC/ec \sim 5 \times 10^{12}$

- Stored energy = $E_{\text{SR}} = \frac{IC}{ec} \times e\mathcal{E} [\text{eV}] \sim 3000 \text{ J}$



Stored energy @ LHC: 27 km, 6.5 TeV, 0.6 A \Rightarrow 350 MJ!!



What about the energy stored in a storage room? Let's assume a current of about half an amp, and a circumference of half a kilometre. Obviously, the time taken for a given electron to execute a single orbit of the ring is to a very high degree of accuracy, simply, big " C " divided by little " c " or the order of 2 microseconds. So the number of columns of charge in the ring is simply " I " multiplied by big " C " divided by little " c ", and hence the number of electrons in the ring is " I " times big " C " divided by " e " little " c ", which is of the order of 5 times 10 to the 12 electrons. Each of these electrons has a kinetic energy of the storage ring energy or little " e " times the energy in electronvolts. The total stored energy is thus of the order of 3 kilojoules, or the amount of heat required to vaporise a thimble full of water. It's an interesting exercise to carry out the same simple calculation for the Large Hadron Collider at CERN. In this case, however, the stored energy is some 350 megajoules, or about the kinetic energy energy of a jumbo jet in mid-flight.

Notes

Summary



2m 57s

Radiated power from bending magnets

- Energy loss of all electrons per single orbit around the ring
 - $= P \times C/c \sim 1 \text{ J}$
- Energy loss per electron per orbit
 - $= (P \times C/c) \times (ec/IC) = Pe/I \sim 1 \text{ MeV}$

From our knowledge of the radiative power output from a typical storage ring, we can immediately calculate the energy loss of the electrons. The energy loss of all the electrons in a single turn around the ring is simply the power output times the time needed to execute a single orbit, or " P " times big " C " divided by little " c ", which is of the order of 1 joule. In order to determine the energy lost per electron, we must divide by the number of electrons in the ring, which we have already determined to be equal to " e " times little " c " divided by " I " times big " C ". This yields an energy loss per electron per turn of the order of 1 megaelectron volt. This is typically still less than one part in a thousand of the electron's kinetic energy. But this loss happens every 1 or 2 microseconds, so left uncorrected, the electrons would lose most of their energy well within a couple of milliseconds.

Notes

Summary



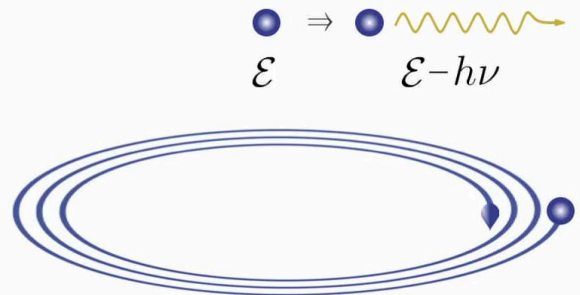
4m 27s

RF cavity – the beating heart of a synchrotron

- Total radiated power (BMs + IDs) ~ 0.5 MW
- \Rightarrow electrons lose $P_{e/I} \sim 1$ MeV/turn (~ 0.03 % of \mathcal{E} , or all of \mathcal{E} in 6 ms)
- Exponential decay with time of electron energy in storage ring
- Remember

$$\rho[\text{m}] = 3.3 \frac{\mathcal{E} [\text{GeV}]}{B [\text{T}]}$$

- Electrons would spiral in from reference orbit and be lost
- How to avoid this?



Long before an electron would expend its kinetic energy in the form of emitted radiation, it would leave its stable orbit due to a mismatch of the dipole bending radius and its magnetic field strength. Not to mention problems with other correcting magnets such as the quads and sextupoles discussed in more detail later. The electron would leave the stable orbit and crash into the vacuum pipe walls producing a cascade of [inaudible 00:06:12] Needless to say, such a scenario must be avoided. The question is, how?

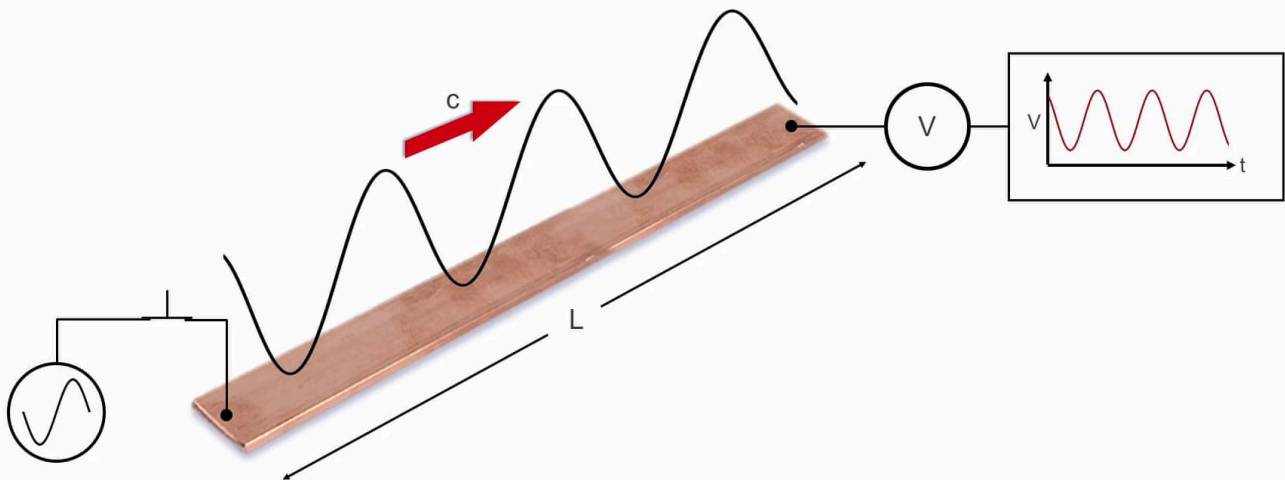
Notes

Summary



5m 39s

Time out – how fast is electricity?



N.B. speed of individual electrons \sim mm/s!!

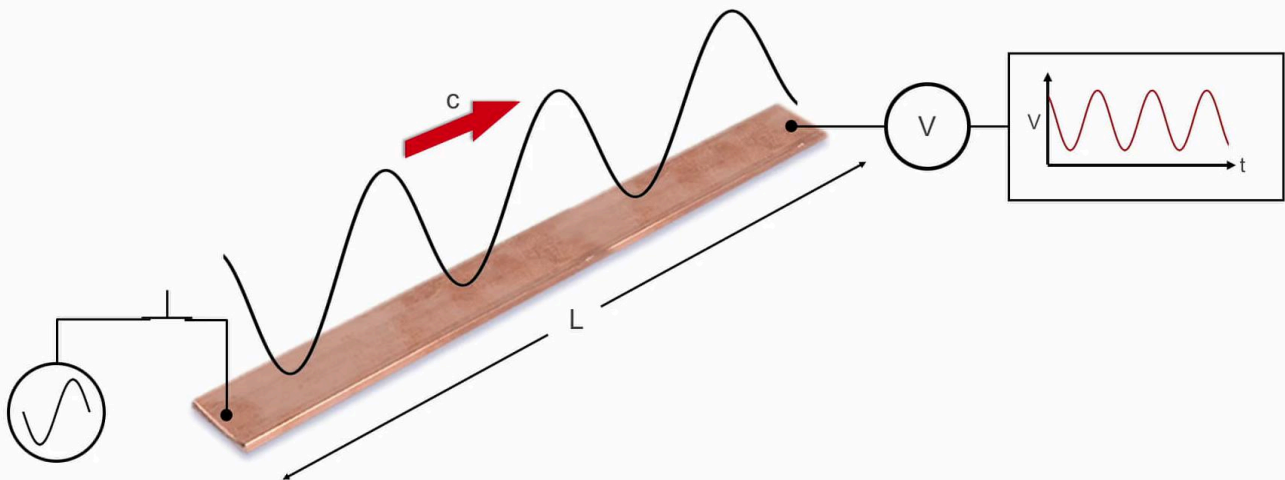
Now, before we answer this, we need to ask ourselves a simple question. How fast is electricity? Let's consider a bar of conducting material of length " L ". A voltage supply with a switch is attached at one end, while at the opposite end, the voltage is recorded as a function of time. At exactly the same time, the switch is thrown to provide the voltage and the recording of the voltage begins at the other end. The volt metre, or oscilloscope will record the arrival of the voltage after a time, " L " divided by " c ". In other words, the electrical field moves down the conductor at the speed of light. If " L " is 1 metre, " L " divided by " c " is 3.3 nanoseconds. Note, by the way, this speed does not correspond to the speed of individual electrons which travel at drift velocities measured in millimetres per second. The recorded speed is that of the propagating electromagnetic wave. Now, if we replace the DC voltage source with an oscillatory supply, this variation in potential will propagate down the conductor with the speed of light. If the periodicity of this oscillatory source is similar to, or shorter than " L " divided by " c ", there will be a clear difference in the electrical potential along the length of the conducting bar.

Notes

Summary



Time out – how fast is electricity?



N.B. speed of individual electrons \sim mm/s!!

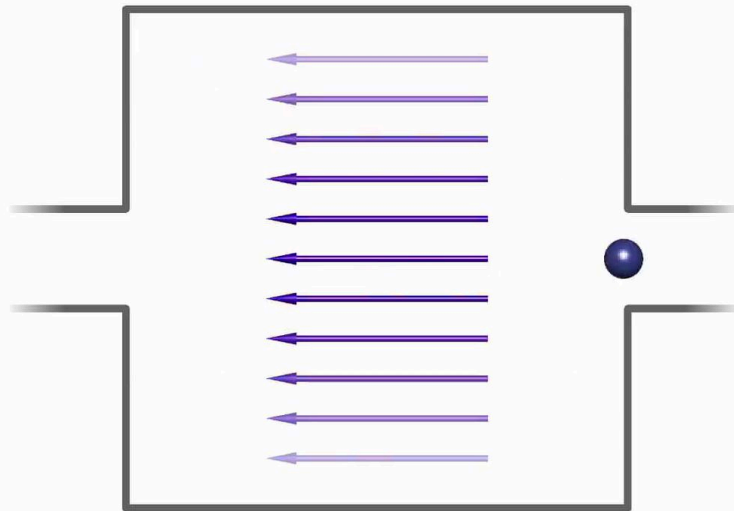
Note that a periodicity of 3.3 nanoseconds corresponds to a frequency of 300 megahertz, which sits squarely in the radio frequency range of the electromagnetic spectrum.

Notes

Summary



RF cavity – the heart of a synchrotron



$$F_E = -eE$$

Okay, now let's consider a cavity shaped like a pillbox with a length "L" similar to, or somewhat shorter than "c" times "t" where "t" is equal to 1 upon the frequency of the periodicity of a connected radio frequency power supply. The opposite ends of the cavity have openings to vacuum pipes with diameters much smaller than the wavelength of the radio waves 1 upon "ct". This means that these radio waves cannot significantly propagate down the vacuum pipes because their wavelength is too large. Consider now an electron entering the cavity from the vacuum pipe. It will experience a varying electric field within the cavity due to differences in voltage at different positions of the conducting electric walls of that cavity. If it enters at a certain moment of the radiofrequency cycle, it will be accelerated. Now, remember, by the way, that the acceleration of the negatively charged electron occurs when the electric field lines are pointing in the opposite direction to the electron's motion. Otherwise, the electron will be decelerated. It should be noted that the cartoon shown here is not accurate in one particular aspect, It shows the electron speeding up significantly, which at the highly relativistic energies of storage of electrons, is not the case.

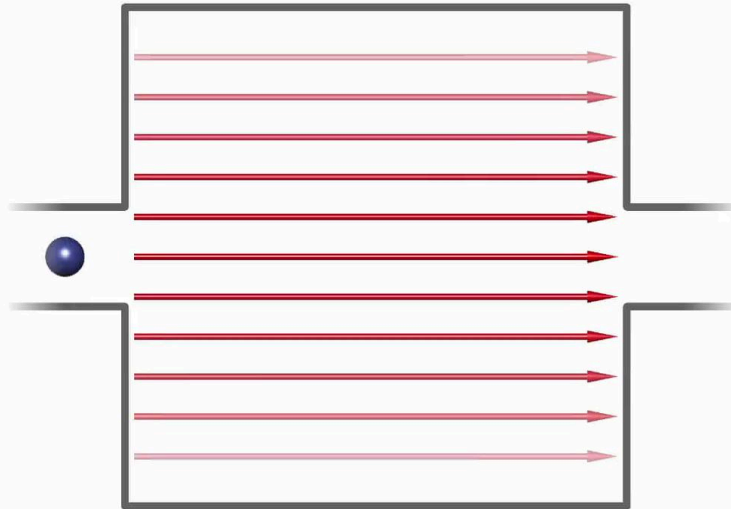
Notes

Summary



8m 19s

RF cavity – the heart of a synchrotron



$$F_E = -eE$$

The actual fractional increase in velocity is extremely small. Instead, relativity affects mean that the electron increases in mass, at least as observed in the frame of reference of the RF cavity. But I didn't really know how to show that in the cartoon and any suggestions from you are more than welcome.

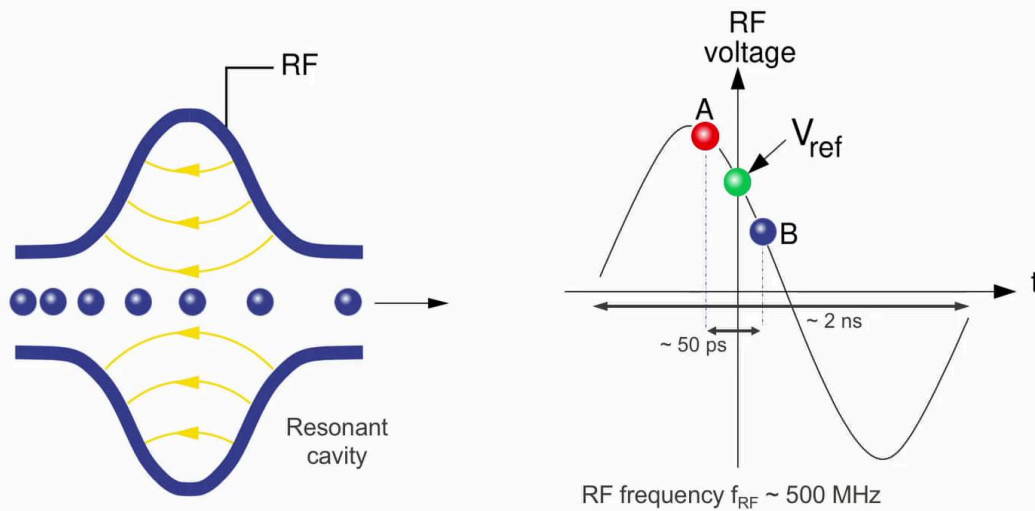
Notes

Summary



10m 01s

RF cavity – the heart of a synchrotron



- Cavities cooled, or even superconducting
- Typically 2 – 4 RF cavities distributed around the ring, each with amplitudes of several hundred keV



Okay, now let's consider two electrons. One of which, "A" has a little less energy than the other one, "B" due perhaps to the fact that it has emitted a couple more photons in the last circuit around the storage room. Electron "A" actually enters the cavity first, for reasons I will explain in just a moment. Even though it has less energy than electron "B". Electron "A" is accelerated by the cavity's field. Electron "B" enters a little later at which point the field amplitude has decreased in its sinusoidal cycle by an amount just enough to accelerate it to the same energy as electron "A" been accelerated. If the electrons that entered the cavity at a significantly different part of the RF cycle, the acceleration or indeed deceleration they experienced would have caused them to become still less stable in their orbit and they would soon be lost. The range of times within a given RF cycle that lead to the correct acceleration can be very narrow, approximately one percent of the cycle period, or a few tens of picoseconds. Any given synchrotron will have typically between two and five RF cavities, storage rings have multiple harp's.

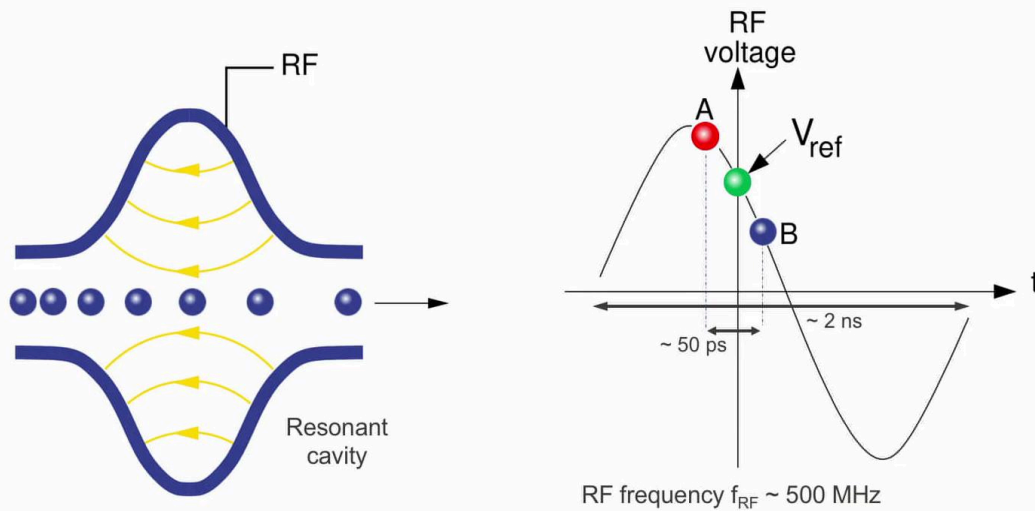
Notes

Summary



10m 23s

RF cavity – the heart of a synchrotron



- Cavities cooled, or even superconducting
- Typically 2 – 4 RF cavities distributed around the ring, each with amplitudes of several hundred keV



From this, it should be apparent that only a very particular time window of the RF cycle is suited to bringing the electrons back to the nominal storage ring energy. As a consequence, the electron current in a synchrotron is not continuous, but instead is composed of bunches, each separated by $1/\text{f}$, the period of the RF cycle, and having a temporal width of approximately one one-hundredth of their separation.

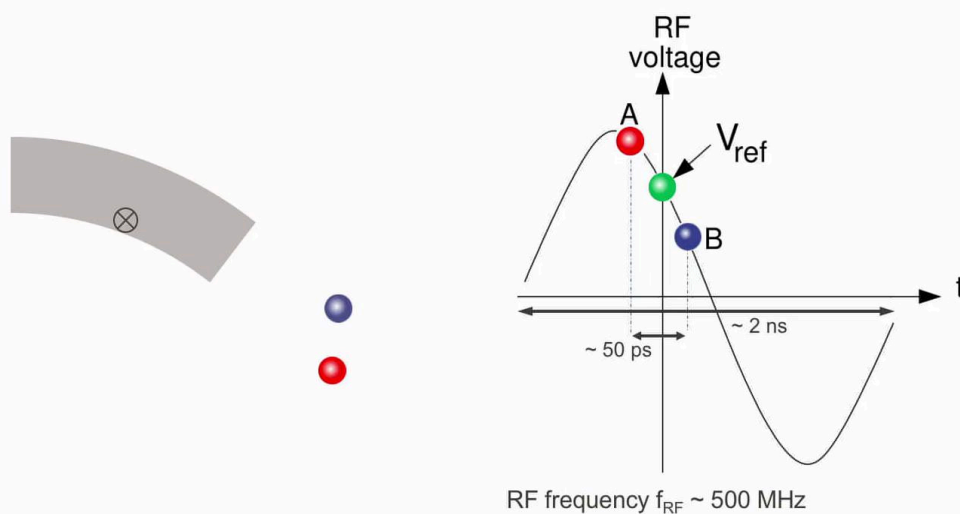
Notes

Summary



11m 59s

RF cavity – the heart of a synchrotron



- Note: slow electrons arrive *earlier* than fast electrons
- $\delta p \propto \delta E \sim 0.1 \%$; $v_{slow} \simeq v_{fast} \simeq c$

So coming back to this question about why "A" gets there before "C", why does this less energetic electron, A, enter the cavity before the more energetic electron B? The difference in their speeds is extremely small, as they are both highly relativistic, but the lower energy electron has a lower mass and would be turned through a smaller bending radius row for a given dipole strength. As the distance covered is proportional to row, electron "A" emerges from the bend arc earlier than electron "B" and this is the reason why the less energetic electron arrives first.

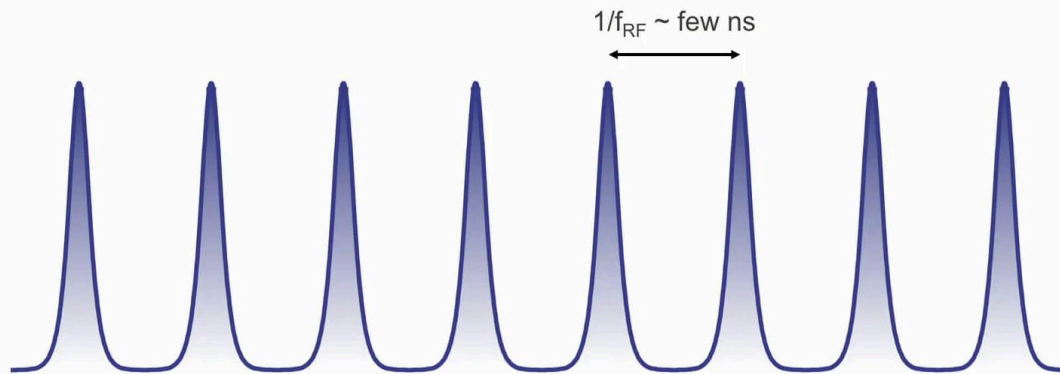
Notes

Summary



12m 34s

RF bunching



$$\begin{aligned} \# \text{ electrons/bunch} &= I/(f_{\text{RF}}e) \sim 0.5/(5 \times 10^8 \times 1.6 \times 10^{-19}) = 6 \times 10^9 \\ \# \text{ bunches} &= f_{\text{RF}} \times (C/c) \sim 300 \end{aligned}$$

So in conclusion, the precise moment required for an electron to enter a cavity in order to be accelerated by the correct amount means that the electron current isn't continuous, but instead consists of bunches separated in time by the RF cavity's electric field periodicity of the order of a few nanoseconds. Each bunch is only a few tens of picoseconds long. The number of electrons in each bunch is simply the current "I" divided by the RF frequency and the elementary charge "e" and amounts to a few billion electrons per bunch. The number of bunches in the ring is simply the RF frequency multiplied by the time to execute a single full orbit, big "C" divided by little "c" and is measured in hundreds of bunches.

Notes

Summary



13m 21s

In the next video...



In the next and final video of this week, we will first look at two of the most important magnetic correcting elements of the magnet lattice, namely the quadrupole and the sextupole, which will lay the foundations to understand so-called bend achromats. We begin with the simplest, double bend achromat, before providing a holistic description of multi bend achromats. These are the development in magnet lattice technology, which has allowed us to advance to the next generation of storage rings, namely, diffraction-limited storage rings.

Notes

Summary

14m 25s

